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MIDCOURSE GUIDANCE USING RADAR TRACKING AND ON-BOARD OBSERVATION DATA

by Gerald L. Smith and Eleanor V. Harper

Ames Research Center
Moffet Field, Calif.

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SUMMARY

The Wiener-Kalman optimal filtering technique is applied to the problem of utilizing combined radar tracking (range and range rate) data and on-board observations for estimating the trajectory of a space vehicle and for guiding the vehicle in cislunar space. Uncertainties included in the problem are bias and nonwhite noise in the radar measurements, radar station location errors, station clock errors, and the error in the knowledge of the velocity of light. The optimal use of the data then provides estimates of the position and velocity state variables and also of the uncertainties.

A digital computer program was prepared for computing the second-order statistics of the estimation errors for a specified schedule of observations. Computations associated with impulsive velocity corrections are also included, and the statistics of guidance performance can be obtained.

Results obtained show that, in general, radar data are superior to on-board observations for estimating the trajectory. However, the increased accuracy does not greatly enhance over-all guidance performance compared to that attainable with only on-board observations. This is because by using on-board observations alone the trajectory can be determined more accurately than it can be controlled, the performance in achieving desired end conditions being ultimately limited by errors in the mechanization of midcourse maneuvers. The results also show that significant improvement can be achieved in the knowledge of radar station locations and the velocity of light by the optimal reduction of radar tracking data.

INTRODUCTION

In references 1, 2, and 3 studies were reported of an on-board midcourse guidance system employing Wiener-Kalman filtering in the trajectory-estimation aspect of the system. In these studies, tracking data from earth-based radar were ignored as a source of trajectory information in the interest of determining the capability of a self-contained on-board system. The desirability of keeping an independent (albeit minimal) on-board system in constant operation for assuring the success of the mission is evident. However, certainly in a real situation ground-based data would never be disregarded if it were at all reasonable to use it. Therefore, it is important that a quantitative evaluation be made of the performance attainable when both ground tracking and on-board observational data are employed.

It is hypothesized in this paper that the raw data from all sources - that is, from on-board the vehicle and from the ground tracking network - are available at a central location where all the data may be processed in an optimal fashion simultaneously. The location of this computing center (i.e., in the vehicle or someplace on the ground) need not be considered here. Also, it will not be necessary here to discuss the alternatives which exist for the configuration and interrelations of the on-board and ground-based systems. We shall be interested only in the quality of the best guidance information which can be generated from all the available data.

A number of studies have been made by various investigators using radar data (but none in which both radar tracking and on-board observations were included). Since the problem is complex, in such studies fairly drastic simplifying assumptions have usually been made to facilitate obtaining numerical results. In the present study it was desired to avoid as many such assumptions as possible. To handle the complexities, a rather massive digital computer program is required. Thus, a major part of the effort in the study has consisted of constructing a suitable computer program. This program is capable of treating a fairly complex model of error sources, but is limited by the size of the core memory of the machine on which it is run. Nevertheless, significant results have been obtained, which are described in the report.

Another reason for undertaking the present study is to determine what difficulties, if any, might arise in applying the Wiener-Kalman optimal filter technique to a more complicated problem than has heretofore been considered. Both the mass of observational data and the number of variables included in the estimation procedure are much greater than in previously reported applications.

The theory of optimal estimation is not given in this paper since it is felt the subject has already been treated adequately. It will be seen that the application of the theory made here is a straightforward extension of the previous work reported in references 1, 2, and 3. The position and velocity of the vehicle and also the subsidiary uncertainties included in the problem are regarded as stochastic state variables, and the optimal use of observational data provides estimates of all the state variables.

An interesting by-product of the present study is the investigation of the idea of using tracking data as a direct means of measuring, or estimating, a number of parameters other than those of the trajectory itself. For instance, tracking data may be used to survey the tracking stations, to adjust the station clocks, to calibrate the radar biases, and to measure the velocity of light. The results presented here give an idea of how well these tasks might be accomplished.

DESCRIPTION OF THE PROBLEM

The mission assumed for most of the results presented here is a circumlunar flight of 6 days duration, coming within 133 km of the moon. The vehicle is assumed to enter the trajectory over the Atlantic and, also, to land in the Atlantic. The earth track of this trajectory is shown in figure 1. A near-earth satellite flight with a circular orbit at 200 km altitude is also used to illustrate the ability of the estimation system to survey the locations of the radar tracking stations.

For analyzing the guidance performance on the circumlunar mission, a linear prediction fixed-time-of-arrival guidance law is assumed. This type of guidance is described in reference 2.

Observations are assumed to consist of two types: on-board optical observations and earth-based radar tracking measurements. The on-board observations may be either (1) theodolite-type measurements of the direction of the line of sight to the earth or moon, or (2) sextant-type measurements of the angle between a selected star and the center of the earth or moon. The radar measurements are of the range and range-rate type. A network of six tracking stations is assumed, which may be arbitrarily located. The particular station sites assumed for the results given here are shown in figure 1. The radar tracking measurements are assumed to depend on an active transponder on the space vehicle, the transponder having only three channels so that only three stations may track at any one time.

The error model assumed for the on-board observations is the same as that used in references 2 and 3; that is, the standard deviation of the error in measuring an angle (by means of either a theodolite or a sextant) is given by the formula

$$\sigma = \sqrt{(10)^2 + (0.001\ \theta)^2}\ \text{sec arc} \tag{1}$$

where θ is one-half the subtended angle of the observed body (earth or moon) expressed in seconds of arc. Observation errors are assumed uncorrelated from one observation to the next.

In the error model for the range, R, and range-rate, $\dot{R}$, measurements, receiver noise, coherent oscillator instability, quantization and time, τ, measurement errors, and unspecified bias-type errors were assumed to be the principal sources of error. The development of the model was rather rudimentary and not intended to refer to any particular radar system. However, the model is considered more or less representative and furthermore contains a number of parameters which can be changed to simulate an actual radar system. The model assumes three uncoupled error sources contributing to each of the two measurements (range and range rate) for each station. One of these error sources is "white" noise (i.e., measurement errors uncorrelated from one sampling to the next), and the other two are correlated noise with different correlation times. Noise with a short correlation time is termed "colored" noise, and with a long correlation time "bias." For the results given here, the following values of standard deviations and time constants were used:

Range errors

White $\sigma = 5.25$ meters

Colored $\sigma = 7.67\times10^{-5}$ R meter, $\tau = 7.5$ sec

Bias $\sigma = 10$ meters, $\tau = 4\times10^{4}$ sec

Range-rate errors

White $$\sigma^2 = 2\times10^{-13}(1 + 0.03\ \dot{R})^2 R^2 + 6.5\times10^{-8}(1 + 0.03\ \dot{R})^2 R + 2\times10^{-4}(1 + 0.03\ \dot{R})^4 + 2.5\times10^{-4}\ \text{meter/sec}$$

Colored $\sigma = 0.05$ meter/sec, $\tau = 0.2$ sec

Bias $\sigma = 0.05$ meter/sec, $\tau = 4\times10^{4}$ sec

The constants employed in this model may be changed (since they are inputs to the program) to simulate different tracking uncertainties.

Plots of the rms sums of these σ's for an assumed Johannesburg station are shown in figure 2. These curves represent the total rms error in the range and range-rate measurements as functions of time for this particular station tracking the vehicle on the outbound leg of the trajectory.

Also included in the study are uncertainties in station location (three quantities for each station), uncertainty in station clock time (one for each station), and uncertainty in the velocity of light. These uncertainties, a total of 25, are considered as random variables, and the optimal estimation system produces estimates of these quantities along with the estimate of the trajectory. Thus, the system has a potential capability for "surveying" the tracking network and improving the knowledge of the velocity of light.

The 1σ values of the initial uncertainties assumed in the results given here are as follows:

Station location	200 meters in each coordinate
Station clock time	2×10^{-4} sec
Velocity of light	400 meters/sec

Injection errors, assumed to be the same as the initial uncertainties in knowledge of the state, must be assumed in order to construct an initial estimation error covariance matrix. Injection errors assumed for the results reported herein are as follows (rms values):

Altitude	3.2 km, 4.5 m/sec
Range	4.8 km, 1.8 m/sec
Crossrange	1.6 km, 1.3 m/sec

Here, the range coordinate is defined as perpendicular to the radius (or altitude) vector and in the plane of the velocity vector. The crossrange coordinate completes an orthogonal reference frame.

Velocity correction mechanization errors must also be specified, in rms terms, to assess the performance of the guidance system. The errors assumed here are:

Magnitude of correction	1 percent
Direction	1°
Cutoff	0.2 m/sec
Velocity correction measurement	0.01 m/sec

THE COMPUTER PROGRAM

The computer program is written in Fortran for use on the IBM 7094 and requires almost all of the 32,000-word memory of this machine. Although tape could conceivably be used for increased storage to permit expanding the program, this would result in much greater computation times with the present Ames system. The size of the core capacity thus limits the complexity and number of features which can be incorporated in the program. Only the performance statistics (i.e., covariance matrices) are computed; there is no provision for the processing of either real or simulated observational data. Besides the computation of an estimation-error covariance matrix of 55 random variables, there is an integration of a reference trajectory and perturbation equations to obtain transition matrices for linear prediction guidance, and the computation of covariance matrices of midcourse velocity corrections and state deviations from nominal. The integration is by means of a relatively simple fourth-order Runge-Kutta routine. The accuracy of this routine is not sufficient for application to a real data-processing problem (e.g., position error at the moon is 50 km, and upon return to the earth, 1,000 km). However, the accuracy is adequate for the purposes of this study; in effect it is as though we were simulating flight in a gravity field which differs slightly from the actual earth-moon-sun field while still preserving the main properties of this field.

The schedule of measurements and velocity corrections is an input to the program. The schedule specifies when and what type of on-board observations and velocity corrections are to be made, and the periods during which ground-based radar tracking is to be allowed. (Radar tracking might have to be discontinued, for instance, when other operations require vehicle orientation incompatible with maintaining the vehicle antenna alined towards the earth.) In all the schedules used here, it was assumed arbitrarily that there would be no radar tracking during periods of on-board observation and velocity correction activities. During periods of allowed radar measurements, the program provides for determining which stations are in view of the vehicle (the line-of-sight at least 5° above earth horizon and not intercepted by the moon). Measurements are then assumed from the first three in-view stations on the list of six provided as an input. Measurements may be made only at the times of integration steps, these times also being inputs to the program.

RESULTS AND DISCUSSION

Numerical results obtained from the program described above are described in this section in terms of three distinct aspects of the problem: (1) trajectory estimation, (2) guidance, and (3) estimation of the subsidiary uncertainties, such as station location and the velocity of light. Some qualitative results are also discussed regarding the practicability of the Wiener-Kalman sequential data processing technique as applied here.

Trajectory Estimation

Figures 3 to 6 show the time histories of the rms errors in estimating the position and velocity of the vehicle for various observation schedules. In figures 3 and 4 a comparison is shown between the estimation error when only on-board theodolite measurements are made and when on-board, range, and range-rate measurements (at intervals of 10 minutes) are made. The schedule of theodolite observations (THEO) is diagrammed at the top of each figure. Earth observation periods are indicated by vertically barred blocks, and moon observations by dots. The times of scheduled velocity corrections are indicated by arrows labeled V.C. There are a total of 90 theodolite observations and 6 velocity corrections in the schedule used for all the results given here.

The radar observation periods are indicated by the pattern of horizontal bars at the top of each figure. As has been noted, the schedules used here embody the assumption that radar tracking is blacked out during periods of on-board observations.

The data in figures 3 and 4 show that at least an order of magnitude improvement is attained by adding only relatively infrequent samples (every 10 minutes) of range and range-rate tracking data from three stations (Rosman, Johannesburg, and Carnarvon) to the on-board observations.

Figures 5 and 6 give some limited results to show the effect of increased amounts of tracking data. The topmost curve in each figure, repeated from figures 3 and 4, is the rms (position or velocity) estimation error for a three-station tracking network, plus on-board observations, with range and range-rate data sampled every 10 minutes. The second curve shows the improvement when an additional three stations (Hawaii, Houston, and Madrid) are added to the network. (This run is for only the first half of the flight.) The improvement, due to the larger amount of tracking data (510 measurements as compared to 267), is significant but not dramatic. The lower curve shows the improvement for the three-station network when range and range rate are sampled once a minute, or approximately ten times as many data points for a given period of tracking. The improvement is about 40 percent. This run was terminated after 48 hours of flight when 1530 radar data points had been simulated.

All the data shown so far are for situations in which it is assumed that each radar data point includes both a range and a range-rate measurement. The relative importance of these two types of data can be determined by computer runs in which first range rate and then range data are excluded. A comparison of the results obtained for these assumptions is shown in figure 7, which shows the rms position estimation error for the three-station network plus theodolite observation situation. It is seen that with only range information the performance is virtually the same as with both range and range-rate data (cf. fig. 5), and with only range-rate information, performance is roughly three times poorer. The conclusion is that, for the error model assumed, most of the trajectory estimation information is contained in the range measurements, and for all practical purposes range-rate measurements might as well be omitted.

Guidance System Performance

For the assessment of guidance system performance, the rms end-point results and total rms midcourse velocity correction must be obtained. Figure 8 shows the rms error in predicting the end-point miss for the situations of theodolite observations only, and theodolite plus three-station tracking every 10 minutes. The break in each of the curves occurs at perilune because of the change in definition of end-point at this time (i.e., on the out-bound leg the end-point is perilune, and on the return it is vacuum perigee). The difference of roughly an order of magnitude is similar to that shown in figures 3 and 4.

A summary of guidance performance is given in table I for the two end-points, perilune and perigee. Results are shown for three conditions: (1) on-board theodolite observations only (a total of 90 observations); (2) range and range-rate measurements only, using a network of three stations and a 10-minute sampling rate (1107 observations); and (3) combined theodolite and radar measurements (90 on-board and 571 ground observations).

The results show that whereas using radar tracking (condition 2) rather than on-board observations (condition 1) reduces end-point uncertainties by about one or two orders of magnitude, the end-point deviations are controlled only a little less than twice as well. The reason is that the end-point deviation is strongly dependent upon the error in applying the final velocity correction, and this is a function of the control mechanization errors, not the estimation errors. The difference in the total velocity correction required is almost insignificant in the two cases. This indicates that relatively little fuel penalty can be ascribed to the omission of ground-based tracking data.

Condition 3, in which a combination of on-board and range and range-rate data is used, is seen to give slightly poorer results, mostly in regard to the uncertainties, than when range and range-rate data are used alone. This is because there are only about half as many radar observations in this run as a result of the radar black-out assumed during on-board observation periods. It might be pointed out that this assumption is arbitrary and probably unrealistic for most applications.

How far the vehicle departs from its nominal, or precalculated, trajectory during the flight is not an important measure of the system performance. However, this deviation is of some interest when the validity of the linearity assumptions which underlie the estimation equations is considered. The rms position deviation is shown as a function of time in figure 9. (The velocity deviation is of the same character but is not shown.) It is seen that the application of velocity corrections fairly early in the outbound and return legs of the flight is reasonably effective in keeping the deviations within limits. The deviations shown in figure 9 are not considered large enough to produce linearity problems. It is seen that there is not a substantial difference between the conditions of on-board observations only and combined on-board and tracking station observations.

Estimation of Subsidiary Uncertainties

The subsidiary uncertainties considered in this study are errors in the knowledge of the station location, the station clock time, and the velocity of light. Bias errors in the range and range-rate measurements can also be considered in some circumstances to be in the same class, although these are more transient in character since they are, at least in part, affected by adjustments in the "tuning" of the electronic systems.

Other errors which may be equally important in some applications, such as uncertainties in the astrodynamic constants, are not included.

All the results shown here are for situations in which simultaneous range and range-rate data were assumed. However, it should be pointed out that, as in the trajectory estimation situation, most of the information regarding the subsidiary uncertainties comes from the range data, and the range-rate data could be omitted with little effect on the results.

Station location estimation.- Figure 10 shows how the rms uncertainty in the knowledge of the station locations is reduced by means of data from a three-station network tracking the circumlunar vehicle. Samplings of range and range rate at 10-minute intervals are assumed, there being a total of 1107 observations. The uncertainties are seen to be reduced by about 40 to 50 percent.

Similar data for a four-orbit tracking of a near-earth satellite at 200 km altitude are shown in figure 11. A six-station network is assumed, with samplings of range and range rate every minute, for a total of 54 observations.

The data are shown only for the stations for which the poorest and best estimates are obtained. The reduction in uncertainty ranges from 61 percent for Rosman to 88 percent for Johannesburg, the reduction being principally determined by the number of observations made from the specific station. In this run the vehicle never came within view of Madrid and thus no information about this station was obtained.

Obviously, tracking at relatively small ranges is most effective in surveying the stations. For the circumlunar tracking most of the information comes from the early part of the flight while the vehicle is close to the earth; it is also noted that only 54 observations of the near-earth satellite were more effective than the 1107 observations of the circumlunar vehicle. An interesting point which shows up strongly in figure 11 is that information about a particular station is obtained not only from tracking data from that station, but also from data from other stations in the network, provided the first station has already tracked the vehicle. This might be termed a triangulation effect.

Results for the circumlunar vehicle runs in which theodolite observations and a three-station network were assumed are shown in figure 12. The performance in estimating station location is not so good as that in the situation without on-board observations (cf. fig. 10) because there are fewer radar observations, and these obviously contain the most information about station location.

Velocity of light estimation.- In figure 13 is shown the reduction in the uncertainty in the velocity of light achieved by tracking the circumlunar vehicle from a three-station network with 10-minute spacing between measurements. A rather striking improvement of almost 10 to 1 is obtained. The majority of the improvement occurs during the early tracking while both the uncertainty and the range rate are relatively high. A second significant drop in the error occurs just as the vehicle returns to view from behind the moon. The physical reason for this phenomenon is not known at this time.

Similar data for the runs in which theodolite observations were included are shown in figure 14. The performance is not so good as for tracking data alone because of the smaller number of radar data points (267 versus 531 on the outbound leg). However, with a six-station network the total number of radar measurements is restored to 510 on the outbound leg, and the performance is again as good as that without the interference from on-board observations.

In the case of the near-earth satellite, reduction in the velocity of light uncertainty is insignificant. The results are shown in figure 15. Of course, there were only 54 observations in this run, but the major factor appears to be that the range and range rate are much smaller for the near-earth trajectory than for the circumlunar trajectory.

Station clock time estimation.- The station clock time uncertainties proved to be insignificant at the level of 2×10^{-4} second rms. Even at 2×10^{-3} second these uncertainties have little effect on the system accuracy, and the tracking data do not materially improve the estimates of the clock errors. This is illustrated in figure 16 for the three-station tracking of the near-earth satellite.

Radar bias estimation.- In the course of processing the radar data, estimates are produced of the biases on the range and range-rate measurements, along with the estimates of the other uncertainties. The accuracy of these estimates is influenced by the magnitudes and time constants of the biases as illustrated in figures 17 and 18. These figures show the rms errors in the estimates of bias on the range and range-rate measurements, respectively, for the Johannesburg tracking station. The situation is for the three-station network tracking the circumlunar vehicle, plus on-board theodolite observations. Results are shown for (1) the reference bias situation, (2) twice the reference bias, and (3) bias with a time constant ten times the reference value.

For the range bias, figure 17 shows that the system does not do a very good job of estimating the bias, in the sense that the reduction in the bias uncertainty is not significant. This means that relatively little information about range bias is present in the radar data. With increased bias, case (2), the system improves the bias estimate more on a percentage basis than for the reference case, but performance is still not good. With increased time constant, case (3), the estimation performance is poorer than in the reference case.

For the range-rate bias (fig. 18) the estimation performance is better than for the range bias but still not what would be termed good. Both increased bias, case (2), and increased time constant, case (3), result in improved percentage performance, indicating that both these changes increase the range-rate bias information in the radar data. The time histories in figure 18 show that the uncertainty in the bias is reduced sharply during periods of radar tracking from the station concerned and then gradually increases when there is no tracking. This increase occurs because the error model is based on the assumption that there is a slow drift in the bias. When the drift is slower (i.e., a longer time constant), it is seen that the system is able to estimate the bias more accurately because the uncertainty does not increase so much between tracking periods.

Figure 19 shows the effect of differences in the magnitude and time constants of bias errors on the estimation of vehicle position. Increased bias degrades the performance and increased time constant improves performance, but not substantially in either case. The effect of bias on velocity estimation is relatively inconsequential and is, therefore, not shown.

Performance of the Computer Program

Experience with the present program has not as yet given definitive answers to questions regarding the practicability of the Wiener-Kalman sequential data processing technique. However, it can be recorded that no difficulties have been encountered with round-off or other computational problems, even in the handling of 55 stochastic state variables and over a thousand data points.

Running time for the program is found to be determined principally by the number of observation data points. Approximately 1.4 seconds (on the IBM 7094) is required per observation, so that computation time can become fairly lengthy in the processing of a large mass of data. However, considering that this is only a first attempt to produce so complex a program of this type, one may reasonably assume that savings in operating time could be achieved by refinement of the program. Also, some of the operating time is known to be due to certain inefficiencies in the present Ames system and should not be charged against the program.

CONCLUDING REMARKS

It is seen that radar data are generally greatly superior to on-board observations for estimating the trajectory. However, relatively little midcourse correction fuel is saved by using the radar data, and control of end-point conditions is not markedly enhanced. This is due principally to the strong influence of the velocity correction mechanization errors. In other words, for the magnitudes of errors assumed in this study, on-board observations alone contain sufficient information (statistically speaking) to determine the trajectory as accurately as it can be controlled. It might be noted that a slightly different velocity correction schedule, optimized for the situation in which radar data are used, would yield somewhat better performance, but probably not enough better to alter this conclusion.

As to the estimation of the subsidiary uncertainties, the results have shown that optimal use of radar tracking data can be a powerful tool in a wide assortment of subsidiary measurement and calibration tasks. In fact, such tasks can be significant in their own right. Surveying radar station locations, calibrating the radar systems, and measuring the velocity of light more accurately can presumably reduce the uncertainties in these quantities to a level at which they are no longer significant sources of error for subsequent missions.

In the assessment of the value of radar data, it has been seen that range data contain much more information than do range-rate data, at least for the magnitudes of errors assumed in this study. It should, therefore, be seriously questioned as to whether it is worth implementing range-rate measurements when a good ranging radar is available, unless such measurements can be made with much greater accuracy than assumed herein.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Jan. 15, 1964

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2. McLean, John D., Schmidt, Stanley F., and McGee, Leonard A.: Optimal Filtering and Linear Prediction Applied to a Midcourse Navigation System for the Circumlunar Mission. NASA TN D-1208, 1962.

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TABLE I.- SUMMARY OF GUIDANCE PERFORMANCE

Condition (number of observations)	Deviations from nominal		Uncertainties in the estimates		Total corrective velocity, meters/sec
	Position, km	Velocity, meters/sec	Position, km	Velocity, meters/sec	
Results at perilune, rms values					
On-board only (45)	25.9	17.2	4.6	2.7	24.8
Range and range rate only three stations (531)	13.5	9.8	.6	.5	22.6
On-board, range, and range rate (45,267)	13.6	9.9	1.9	1.1	22.6
Results at perigee, rms values					
On-board only (90)	32.5	26.4	18.5	13.0	34.4
Range and range rate only three stations (1107)	18.2	17.2	.2	.2	31.2
On-board, range, and range rate (90,571)	18.3	17.3	.7	.6	31.2

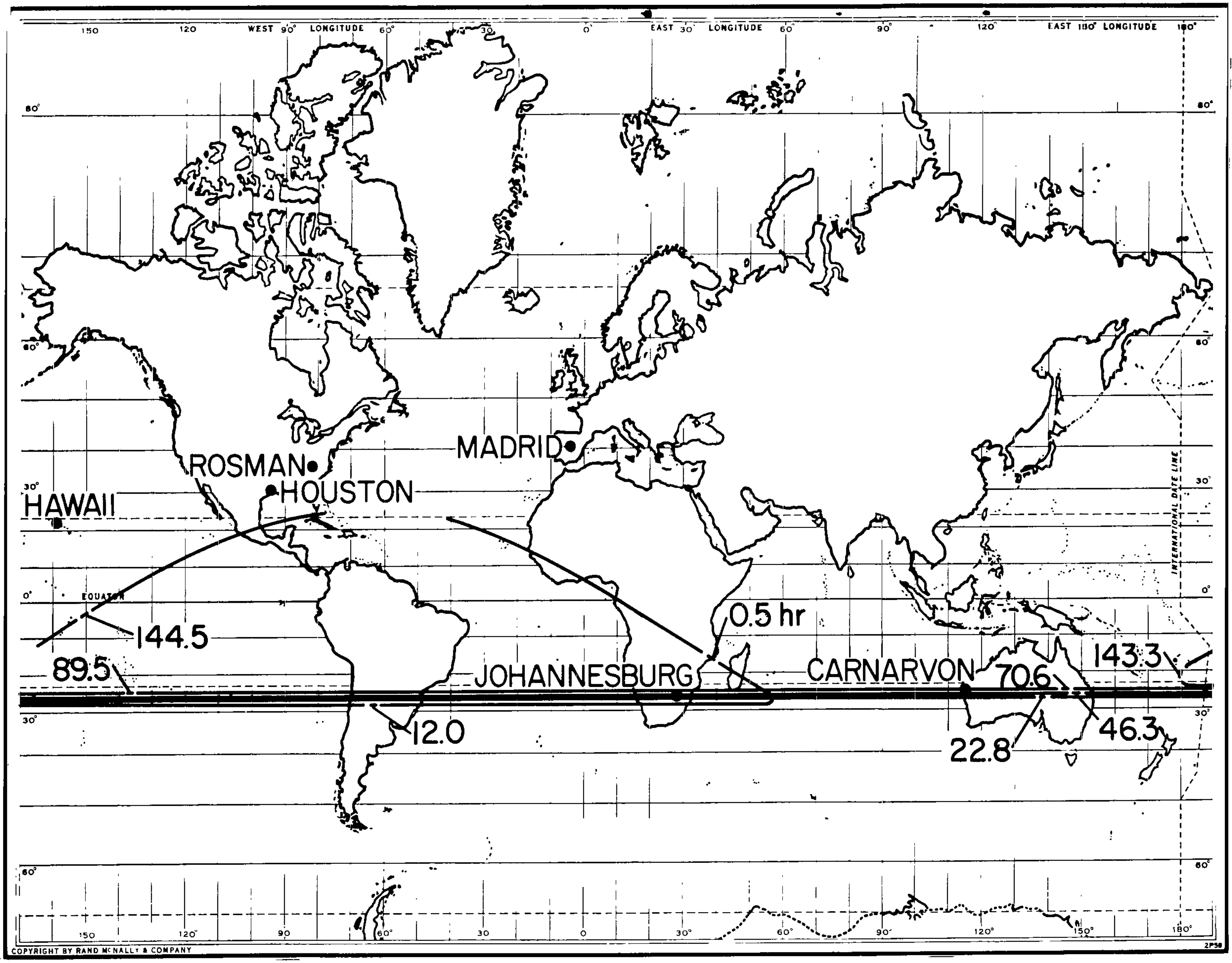

Figure 1.- Radar tracking network and earth track of circumlunar trajectory.

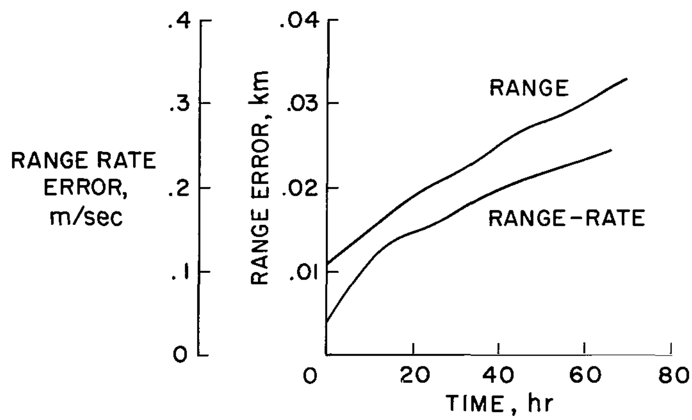

Figure 2.- Radar errors (Johannesburg station).

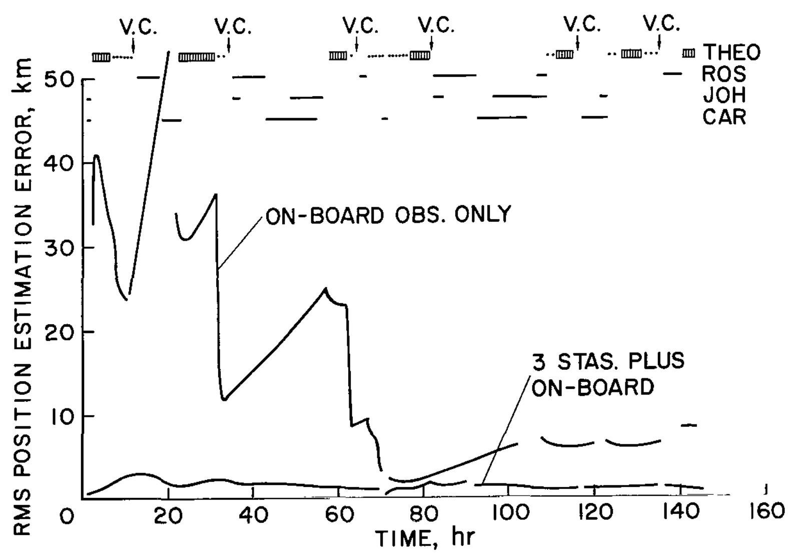

Figure 3.- Position estimation error. Comparison between on-board observations and on-board plus three-station radar tracking.

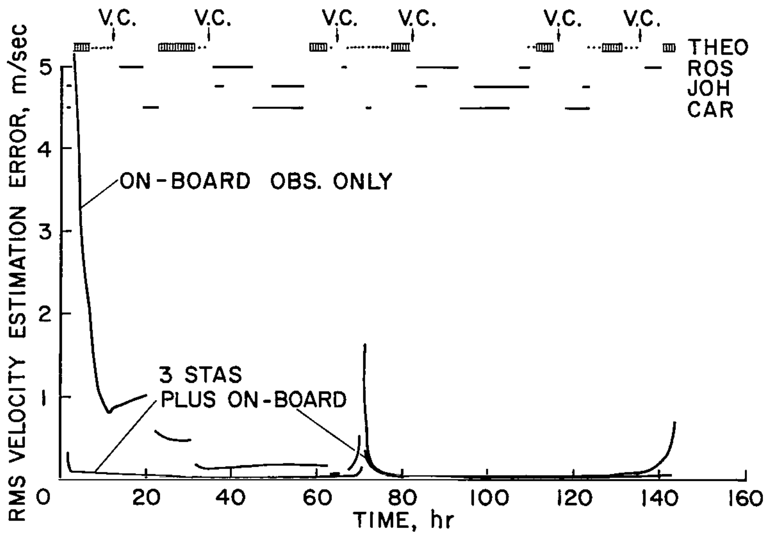

Figure 4.- Velocity estimation error. Comparison between on-board observations and on-board plus three-station radar tracking.

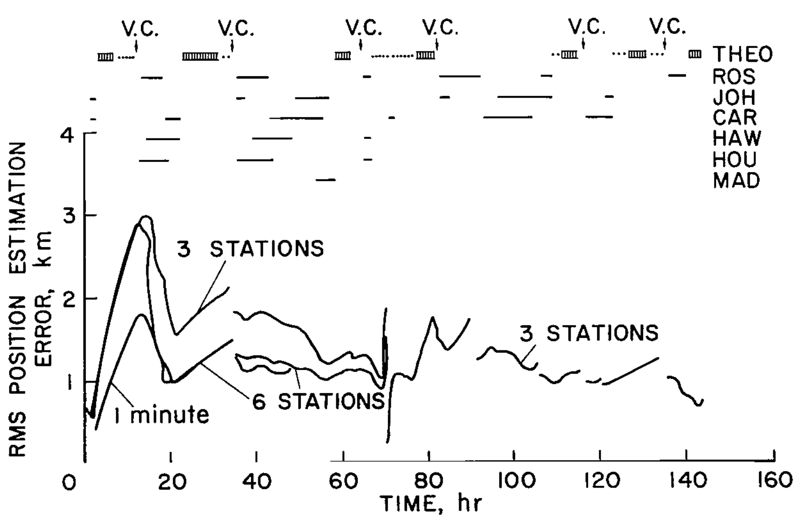

Figure 5.- Position estimation error. The effect of different amounts of radar information.

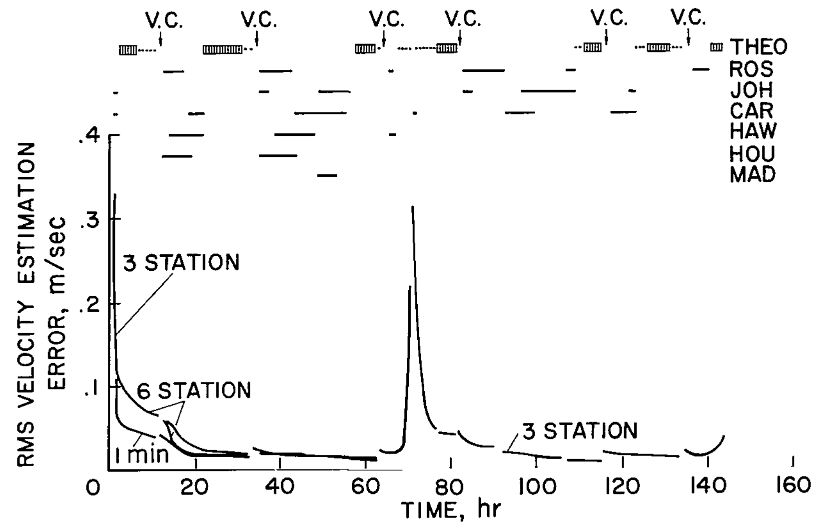

Figure 6.- Velocity estimation error. The effect of different amounts of radar information.

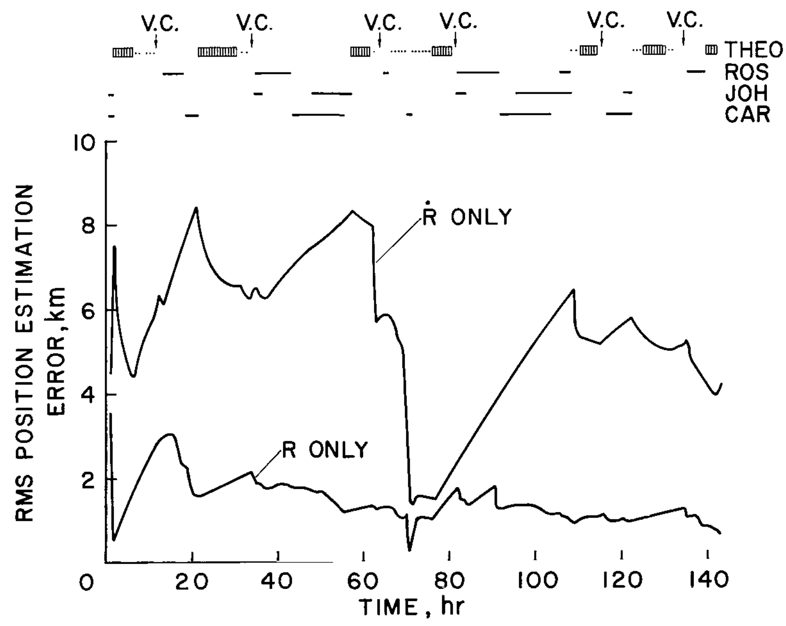

Figure 7.- Relative importance of range and range-rate information.

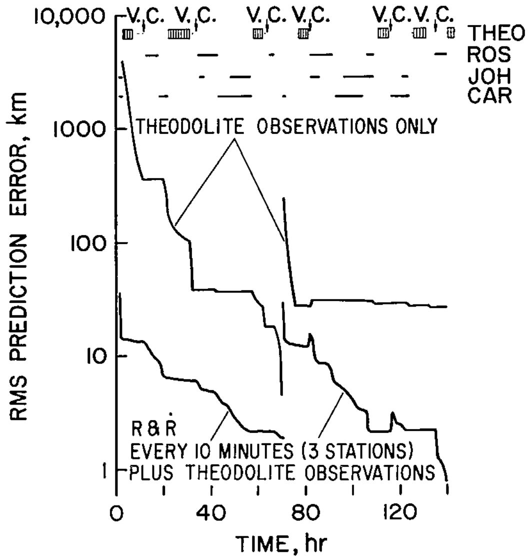

Figure 8.- Error in the prediction of end-point position.

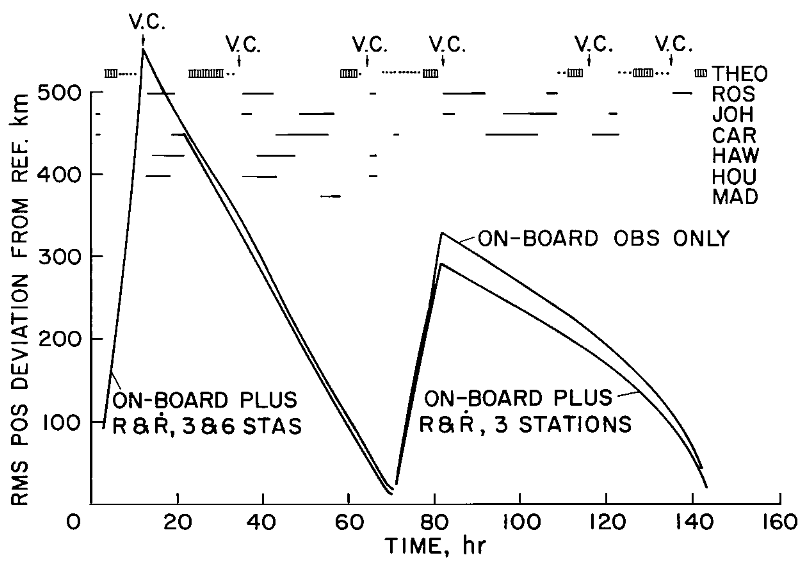

Figure 9.- Deviation from nominal trajectory.

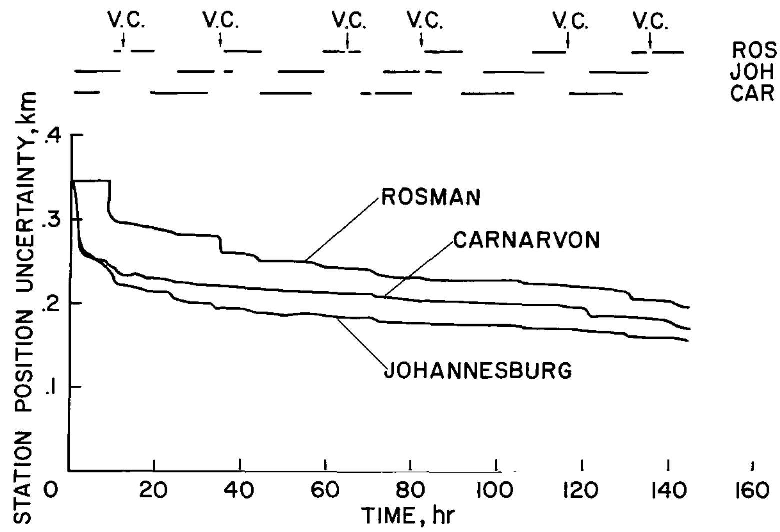

Figure 10.- Estimation of radar station locations from radar tracking of circumlunar vehicle.

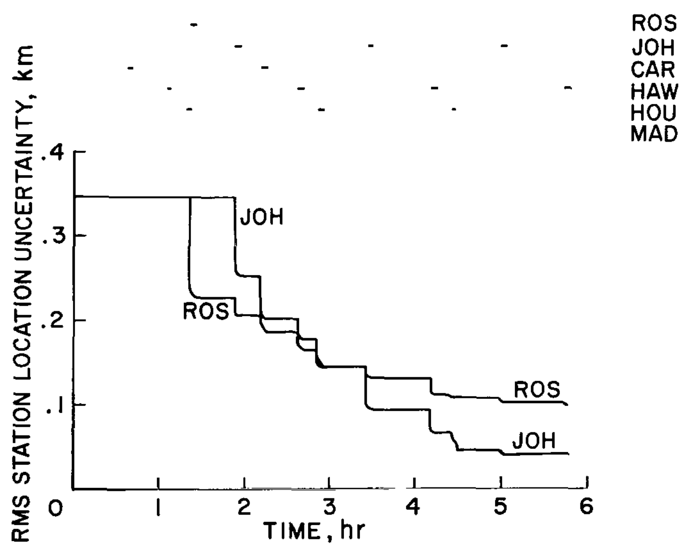

Figure 11.- Estimation of radar station locations from radar tracking of near-earth satellite.

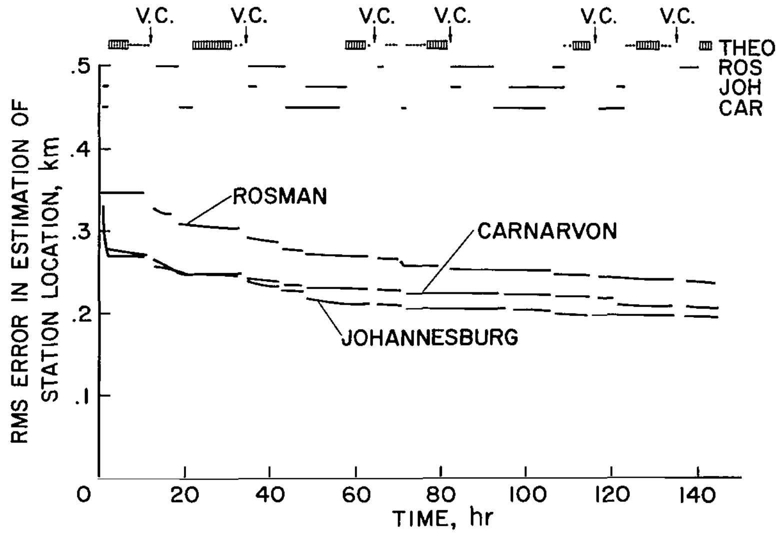

Figure 12.- Estimation of radar station locations from combined radar tracking and on-board observations.

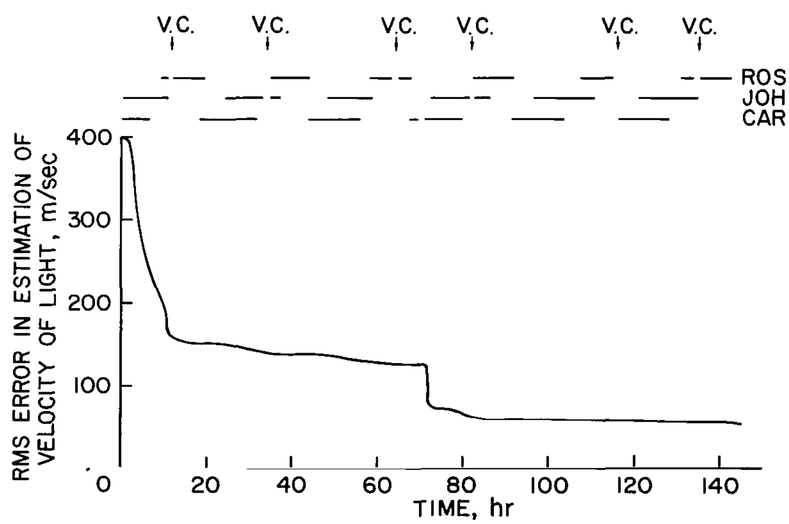

Figure 13.- Estimation of the velocity of light from radar tracking of circumlunar vehicle.

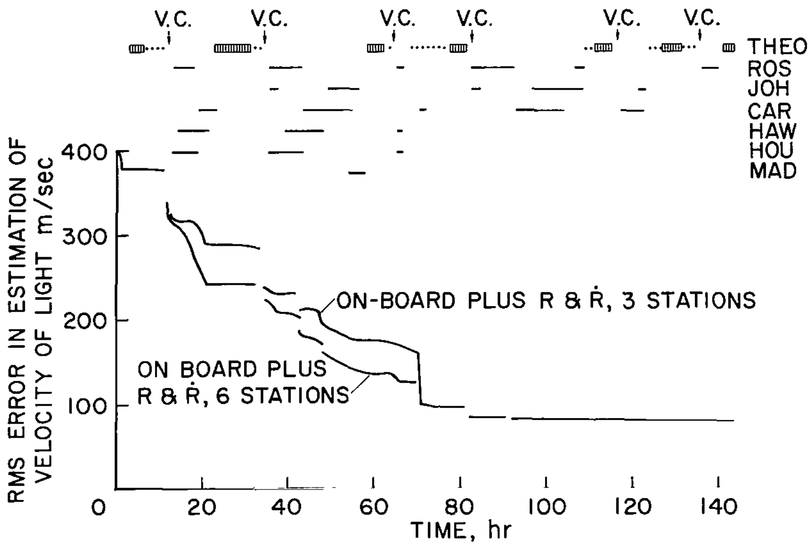

Figure 14.- Estimation of the velocity of light from combined radar tracking and on-board observations.

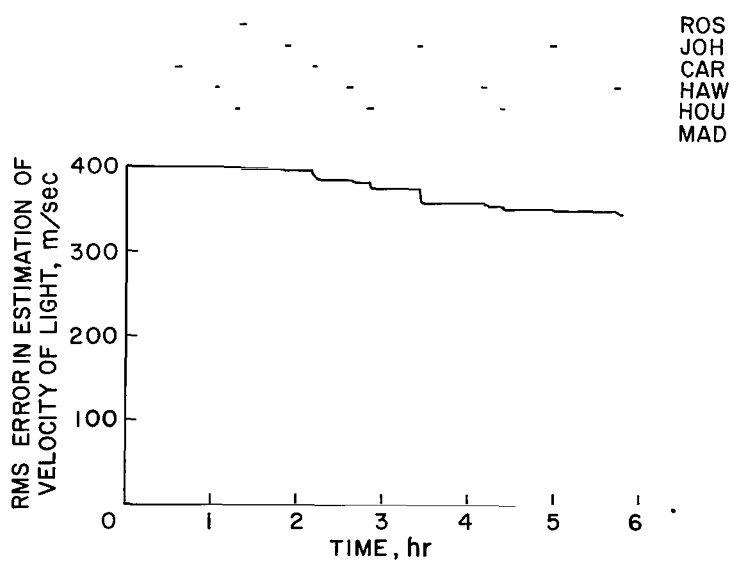

Figure 15.- Estimation of the velocity of light from radar tracking of near-earth satellite.

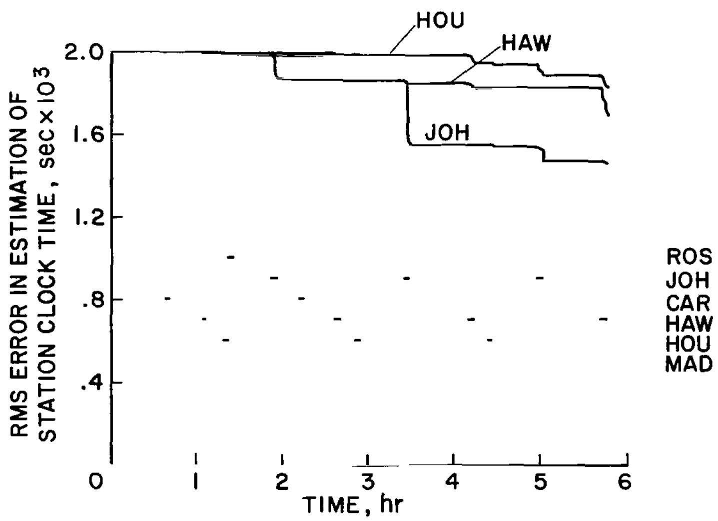

Figure 16.- Estimation of radar station clock time.

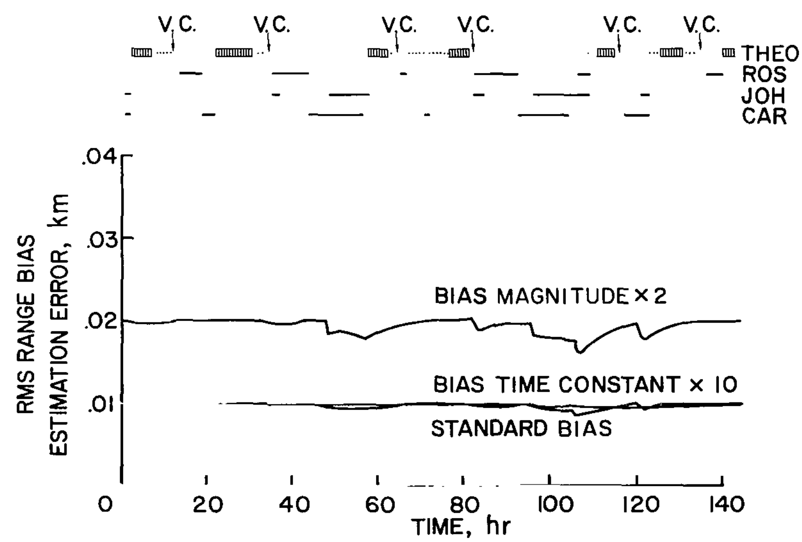

Figure 17.- Estimation of range measurement bias.

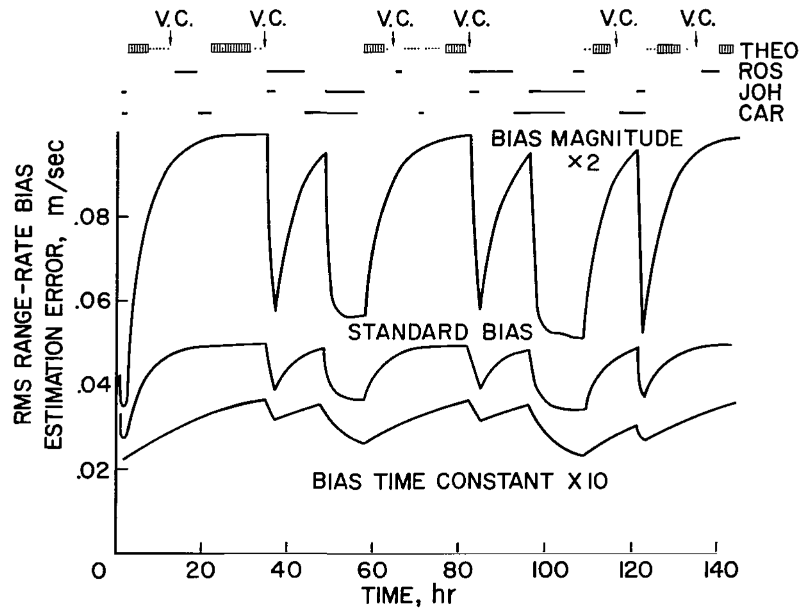

Figure 18.- Estimation of range-rate measurement bias.

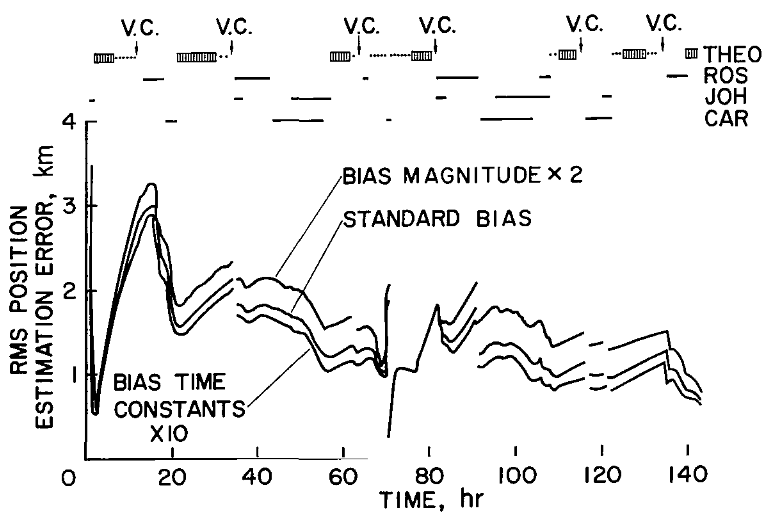

Figure 19.- Effect of radar bias on vehicle position estimation.

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